6.2 Properties of Parallelograms

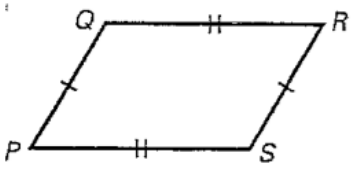
VOCABULARY

Parallelogram a quadrilateral w/ both pairs of opposite sides parallel

THEOREM 6.2

If a quadrilateral is a parallelogram, then its **opposite sides** are congruent.

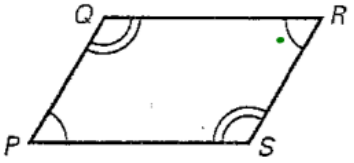
$\overline{QP} \cong \overline{RS}$ and $\overline{SP} \cong \overline{RQ}$



THEOREM 6.3

If a quadrilateral is a parallelogram, then its **opposite angles** are congruent.

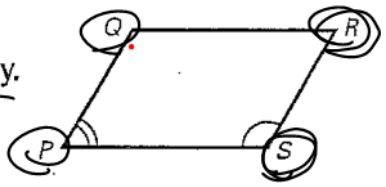
$\angle P \cong \angle R$ and $\angle Q \cong \angle S$



THEOREM 6.4

If a quadrilateral is a parallelogram, then its **consecutive angles** are supplementary.

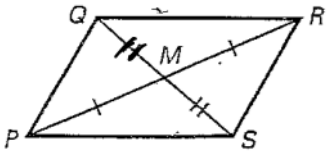
$m\angle P + m\angle Q = 180°$,
$m\angle Q + m\angle R = 180°$,
$m\angle R + m\angle S = 180°$,
$m\angle S + m\angle P = 180°$



THEOREM 6.5

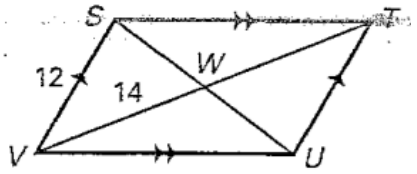
If a quadrilateral is a parallelogram, then its diagonals **bisect** each other.

$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$



Example 1: *Using Properties of Parallelograms*

STUV is a parallelogram. Find the unknown length.



a. *TU* b. *WT*

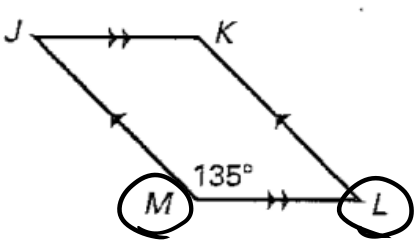
Solution

a. $\overline{TU} \cong \overline{SV}$
$TU = 12$

b. $\overline{WT} \cong \overline{YW}$
$WT = 14$

Example 2: *Using Properties of Parallelograms*

JKLM is a parallelogram. Find $m\angle L$.



Solution

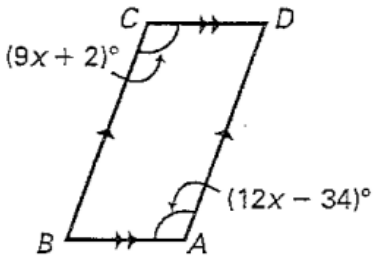
$m\angle L + m\angle M = 180°$

$m\angle L + 135° = 180°$

$m\angle L = 45°$

Example 3: *Using Algebra with Parallelograms*

ABCD is a parallelogram. Find the value of *x.*



Solution

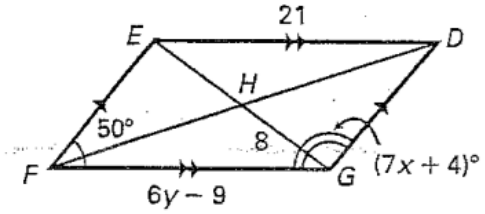
$m\angle C = m\angle A$

$9x + 2 = 12x - 34$

$36 = 3x$

$x = 12$

Use parallelogram *DEFG* to answer the following.

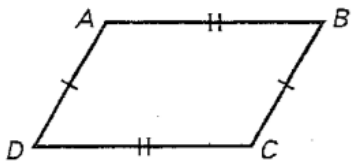


1. Find $m\angle D$. $= m\angle F = 50°$
2. Find *EH*. $= HG = 8$
3. Find y. $6y - 9 = 21 \rightarrow y = 5$
4. Find x. $7x + 4 + 50 = 180 \rightarrow x = 18$

8.3 Proving Quadrilaterals are Parallelograms

THEOREM 6.6

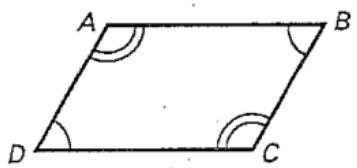
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



ABCD is a parallelogram.

THEOREM 6.7

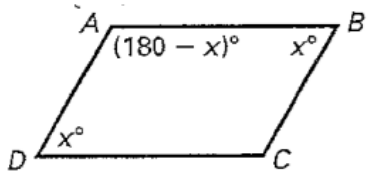
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



ABCD is a parallelogram.

THEOREM 6.8

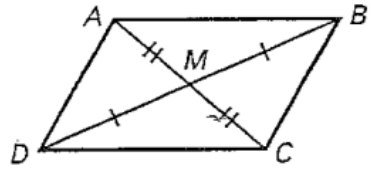
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.



ABCD is a parallelogram.

THEOREM 6.9

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

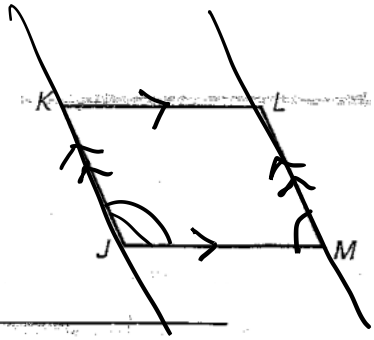


ABCD is a parallelogram.

5.

Example 1: ***Proof of Theorem 6.8***

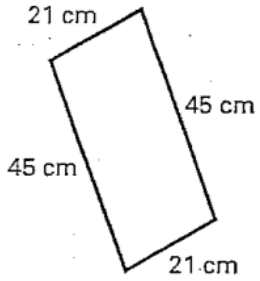
Given: $\angle J$ is supplementary to $\angle K$ and $\angle M$.
Prove: *JKLM* is a parallelogram.



Statements	Reasons
1. $\angle J$ is supplementary to $\angle K$.	1. Given
2. $\overline{JM} \parallel$ $\overline{KL}$	2. Consec. Int $\angle$s = 180° → lines are parallel
3. $\angle J$ is supplementary to $\angle M$.	3. Given
4. $\overline{JK} \parallel$ $\overline{ML}$	4. Consec. Int $\angle$s = 180° → lines are parallel
5. *JKLM* is a parallelogram.	5. Def of parallelogram

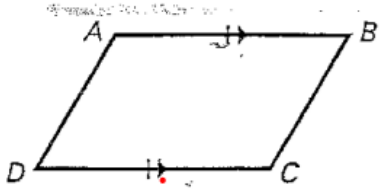
√ A pane in a stained glass window has the shape shown at the right. How do you know that the pane is a parallelogram?

both pairs of opposite sides are congruent



THEOREM 6.10

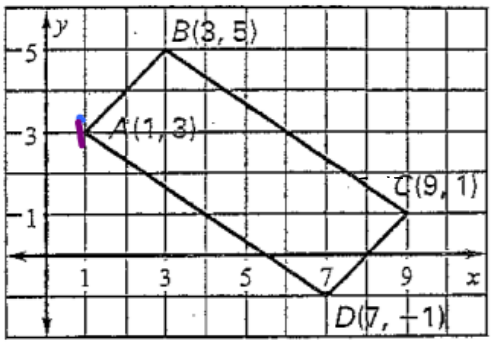
If one pair of opposite sides of a quadrilateral are parallel and $\cong$, then the quadrilateral is a parallelogram.



ABCD is a parallelogram.

Example 2: ***Using Properties of Parallelograms***

Show that $A(1, 3)$, $B(3, 5)$, $C(9, 1)$, and $D(7, -1)$ are the vertices of a parallelogram.



Solution

Method 1 Show that opposite sides have the same slope.

Slope of $\overline{AB}$: $\frac{1}{1} = 1$

Slope of $\overline{BC}$: $-\frac{2}{3}$

Slope of $\overline{CD}$: $\frac{1}{1} = 1$

Slope of $\overline{AD}$: $\frac{2}{-3}$

Method 2 Show that opposite sides have the same length.

$AB = \sqrt{(3 - 1)^2 + (5 - 3)^2} = \sqrt{8} =$ _____

$BC = \sqrt{(9 - 3)^2 + (1 - 5)^2} = \sqrt{52} =$ _____

$CD = \sqrt{(9-7)^2 + (1-(-1))^2} = \sqrt{8}$

$AD = \sqrt{(7-1)^2 + (-1-3)^2} = \sqrt{52}$

Method 3: Show that one pair of opposite sides are congruent and parallel.

Slope of $\overline{AB}$ = Slope of $\overline{CD}$ = __

$AB = CD =$ 2.8

$\overline{AB}$ and $\overline{CD}$ are parallel and $\cong$ So, $ABCD$ is a parallelogram.